Log-Euclidean Metric Learning on Symmetric Positive Definite Manifold with Application to Image Set Classification

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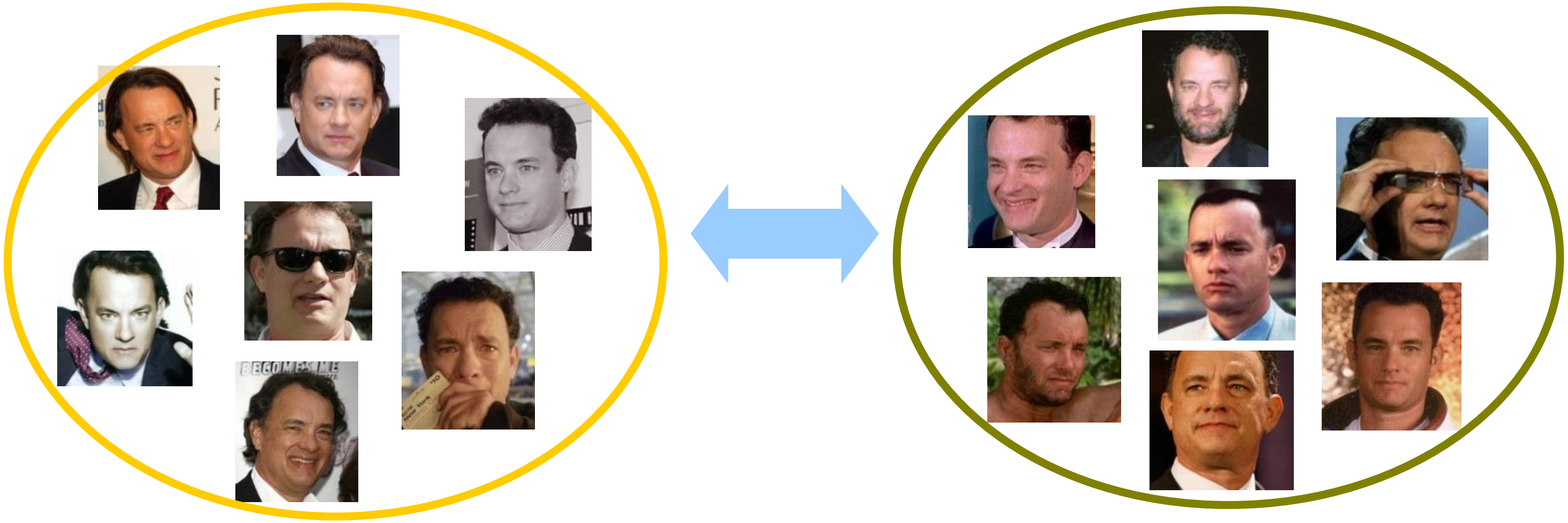
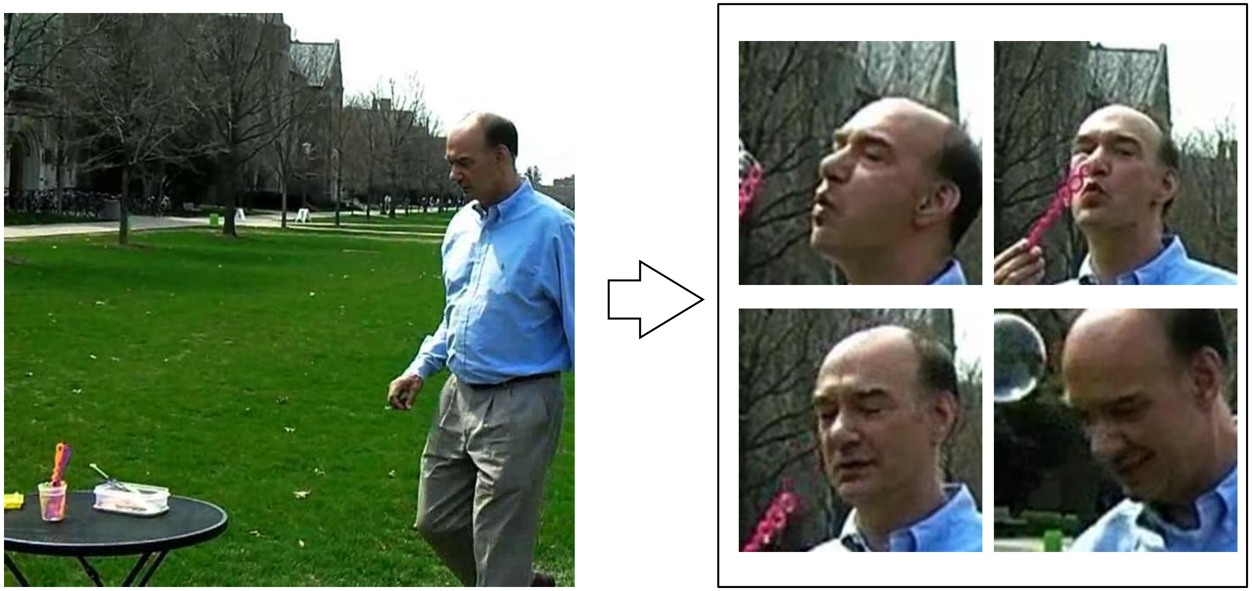
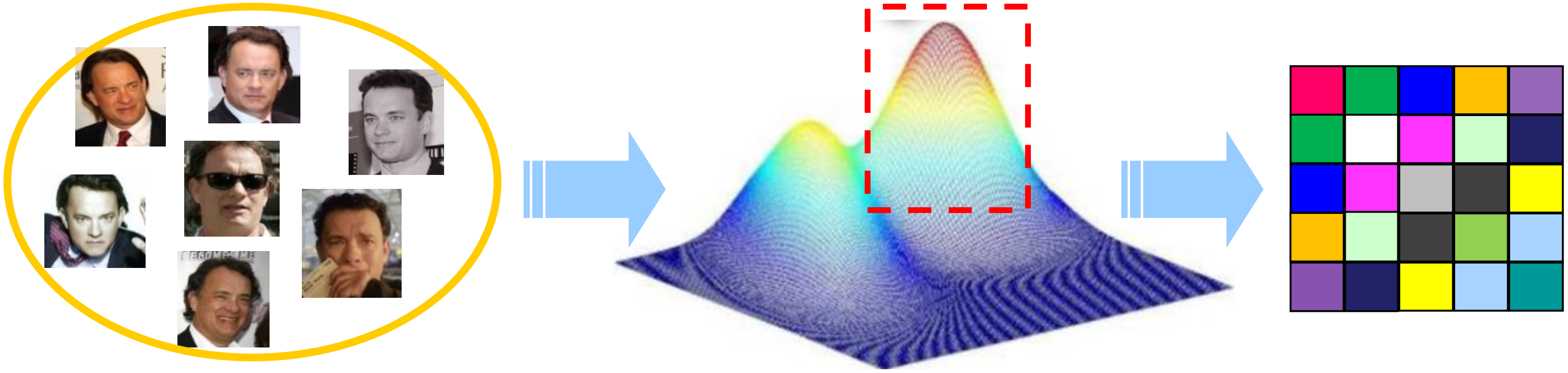
- Training/testing sample is a set of images involving a single subject
  + Rich information to describe subject
  – Complex appearance variations



- Example
  - Video-based face recognition
    - Identify a subject with his/her video sequence
      - Treating video as image set

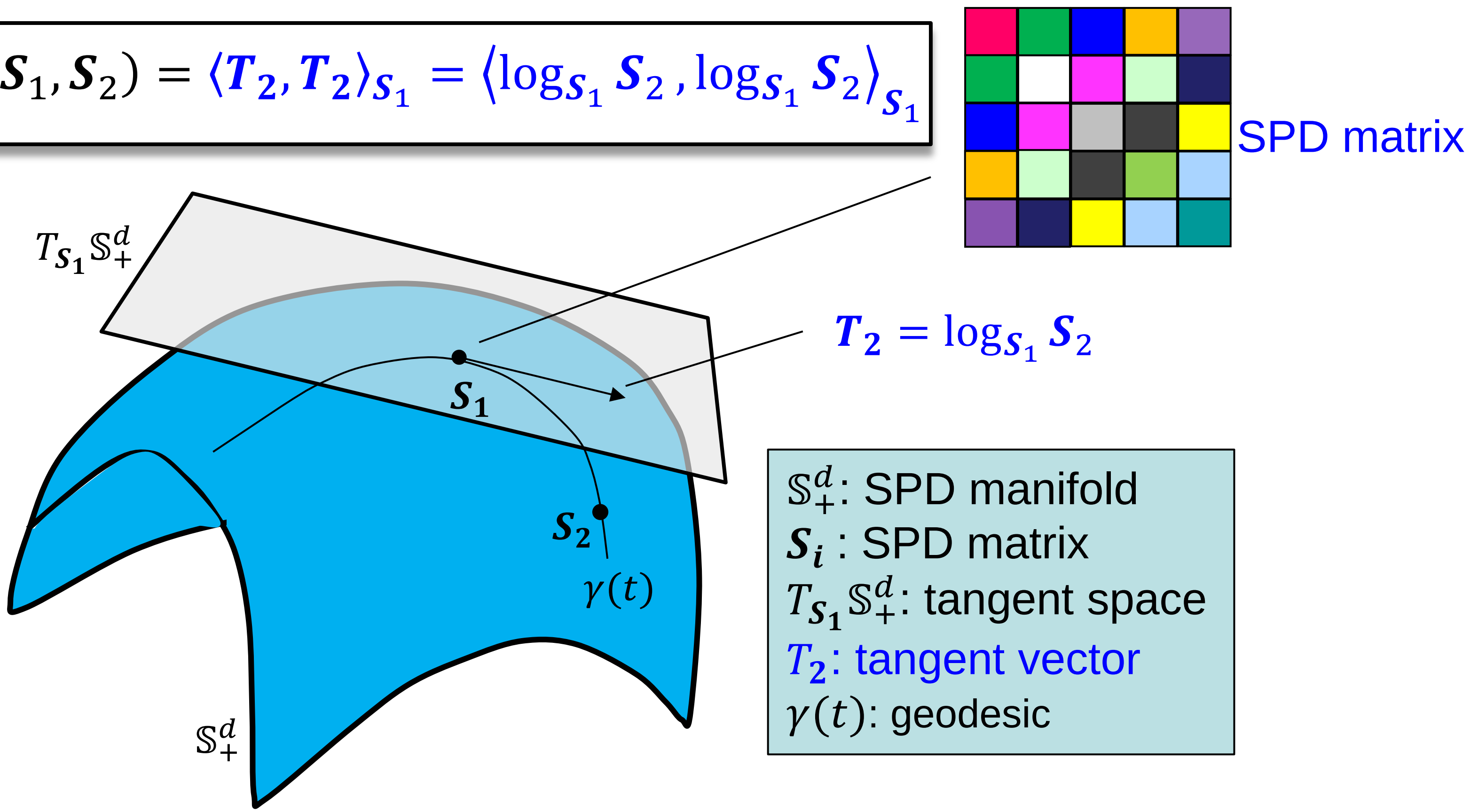


- Represent image set with Gaussian model
  - From Gaussian model to SPD matrix
    - Information geometry theory [Amari & Nagaoka,2000; Lovrić,2000]
      - $\mathcal{N}(x|\tilde{m},\tilde{C}) \sim \boldsymbol{S} = |\tilde{\boldsymbol{C}}|^{-\frac{1}{d+1}} \begin{bmatrix} \tilde{\boldsymbol{C}} + \tilde{\boldsymbol{m}}\tilde{\boldsymbol{m}}^T & \tilde{\boldsymbol{m}} \\ \tilde{\boldsymbol{m}}^T & 1 \end{bmatrix}$
      - $\tilde{\boldsymbol{C}}$: covariance matrix of size $d \times d$, $\tilde{\boldsymbol{m}}$: mean vector of size d



- Account for Riemannian geometry
  - Riemannian metric

$$\delta_g^2(\boldsymbol{S}_1, \boldsymbol{S}_2) = \langle \boldsymbol{T}_2, \boldsymbol{T}_2 \rangle_{\boldsymbol{S}_1} = \langle \log_{\boldsymbol{S}_1} \boldsymbol{S}_2, \log_{\boldsymbol{S}_1} \boldsymbol{S}_2 \rangle_{\boldsymbol{S}_1}$$

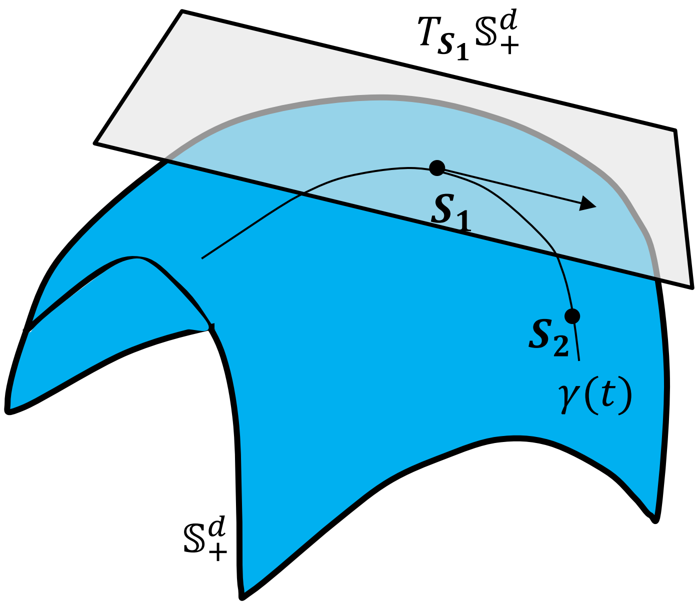


$\mathbb{S}_+^d$: SPD manifold
$\boldsymbol{S}_i$: SPD matrix
$T_{\boldsymbol{S}_1}\mathbb{S}_+^d$: tangent space
$\boldsymbol{T}_2$: tangent vector
$\gamma(t)$: geodesic

- Account for Riemannian geometry
  - Affine-Invariant metric (AIM)

$$d_a^2(\boldsymbol{S}_1,\boldsymbol{S}_2)=\left\langle \log_{\boldsymbol{S}_1}\boldsymbol{S}_2,\log_{\boldsymbol{S}_1}\boldsymbol{S}_2\right\rangle_{\boldsymbol{S}_1}=||\log\left(\boldsymbol{S}_1^{-1/2}\boldsymbol{S}_2\boldsymbol{S}_1^{-1/2}\right)||_{\mathcal{F}}^2$$

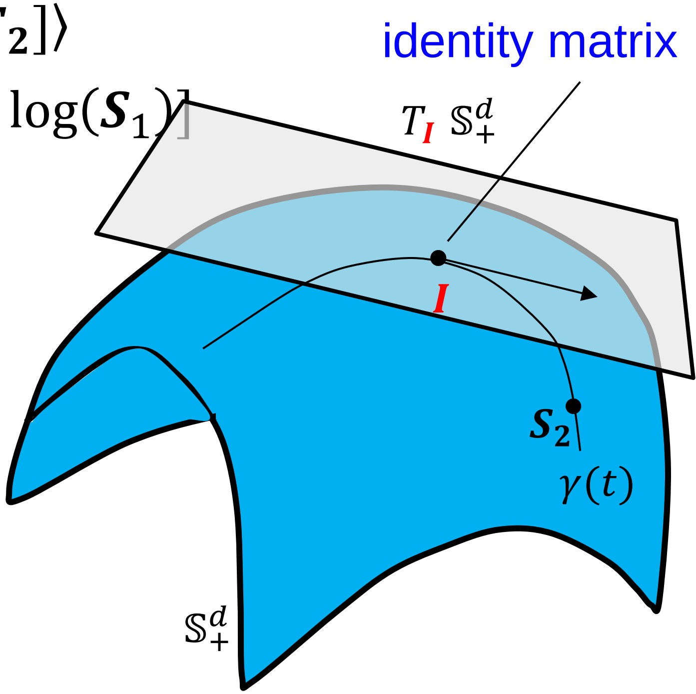
- $\langle \boldsymbol{T}_2,\boldsymbol{T}_2\rangle_{\boldsymbol{S}_1}=\left\langle \boldsymbol{S}_1^{-\frac{1}{2}}\boldsymbol{T}_2\boldsymbol{S}_1^{-\frac{1}{2}},\boldsymbol{S}_1^{-\frac{1}{2}}\boldsymbol{T}_2\boldsymbol{S}_1^{-\frac{1}{2}}\right\rangle$
- $\boldsymbol{T}_2=\log_{\boldsymbol{S}_1}\boldsymbol{S}_2=\boldsymbol{S}_1^{\frac{1}{2}}\log(\boldsymbol{S}_1^{\frac{1}{2}}\boldsymbol{S}_2\boldsymbol{S}_1^{\frac{1}{2}})\boldsymbol{S}_1^{\frac{1}{2}}$
- Computational cost is expensive
  - Main cost: $\log(\boldsymbol{S}_1^{\frac{1}{2}}\boldsymbol{S}_2\boldsymbol{S}_1^{\frac{1}{2}})$

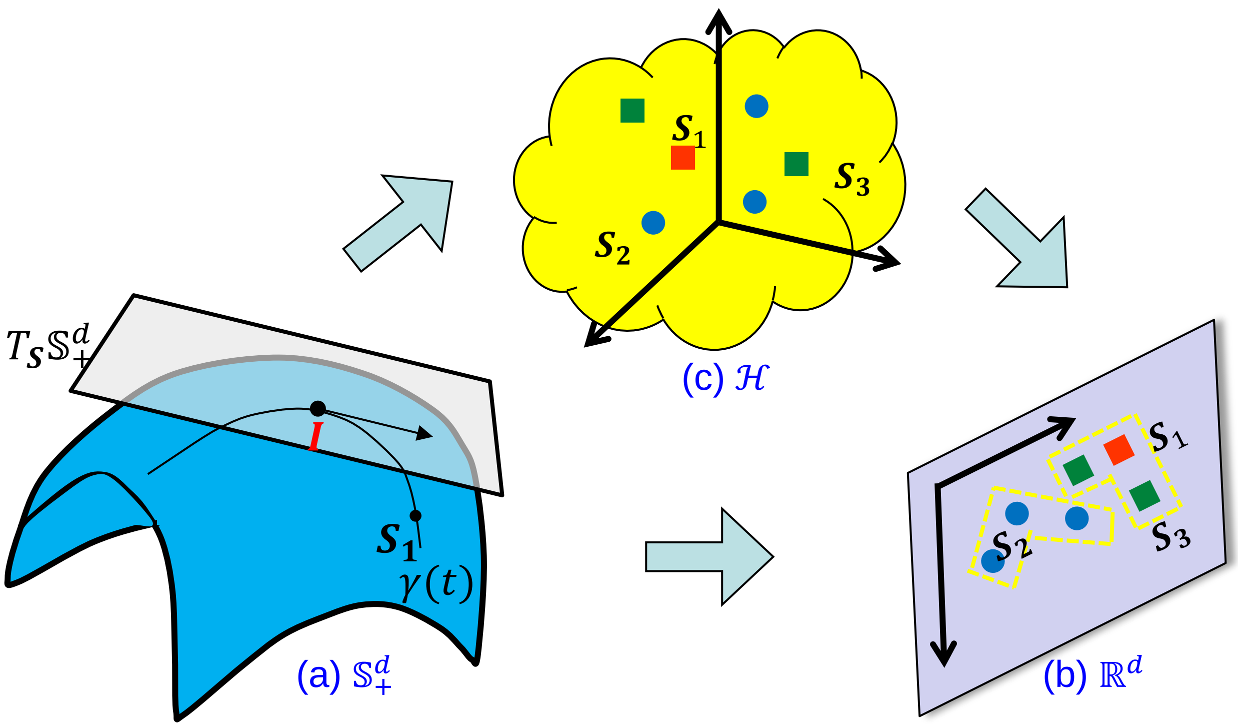


- Account for Riemannian geometry
  - Log-Euclidean metric (LEM)

$$d_l^2(\boldsymbol{S}_1, \boldsymbol{S}_2) = \left\langle \log_{\boldsymbol{S}_1} \boldsymbol{S}_2, \log_{\boldsymbol{S}_1} \boldsymbol{S}_2 \right\rangle_{\boldsymbol{S}_1} = ||\log(\boldsymbol{S}_1) - \log(\boldsymbol{S}_2)||_{\mathcal{F}}^2$$

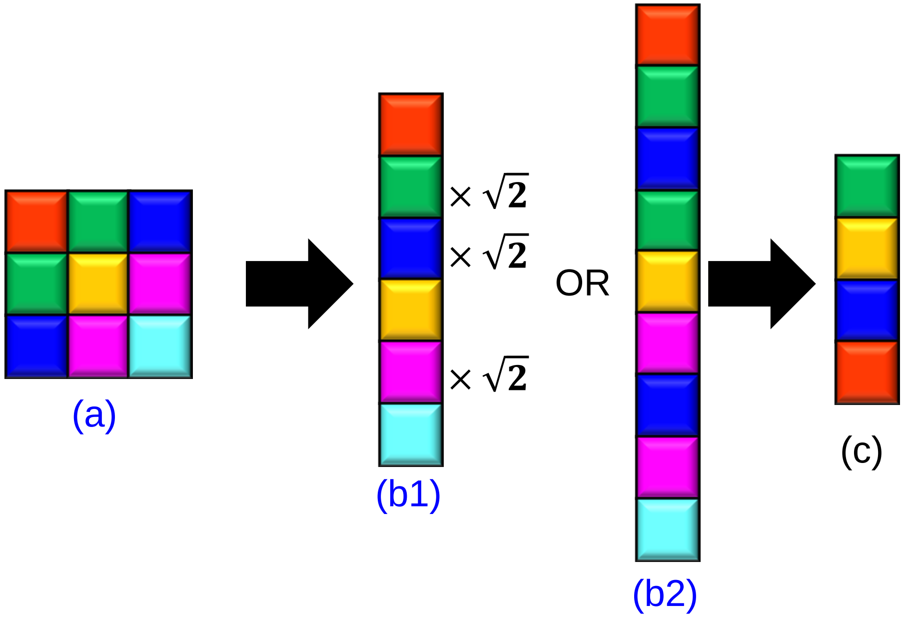
- $\langle \boldsymbol{T}_2, \boldsymbol{T}_2 \rangle_{\boldsymbol{S}_1} = \langle \mathrm{Dlog}(\boldsymbol{S}_1)[\boldsymbol{T}_2], \mathrm{Dlog}(\boldsymbol{S}_1)[\boldsymbol{T}_2] \rangle$
- $\boldsymbol{T}_2 = \log_{\boldsymbol{S}_1} \boldsymbol{S}_2 = \mathrm{D}^{-1}\log(\boldsymbol{S}_1)[\log(\boldsymbol{S}_2) - \log(\boldsymbol{S}_1)]$
- Drastic reduction in computation time
  - Need Euclidean computation in the domain of matrix logarithms



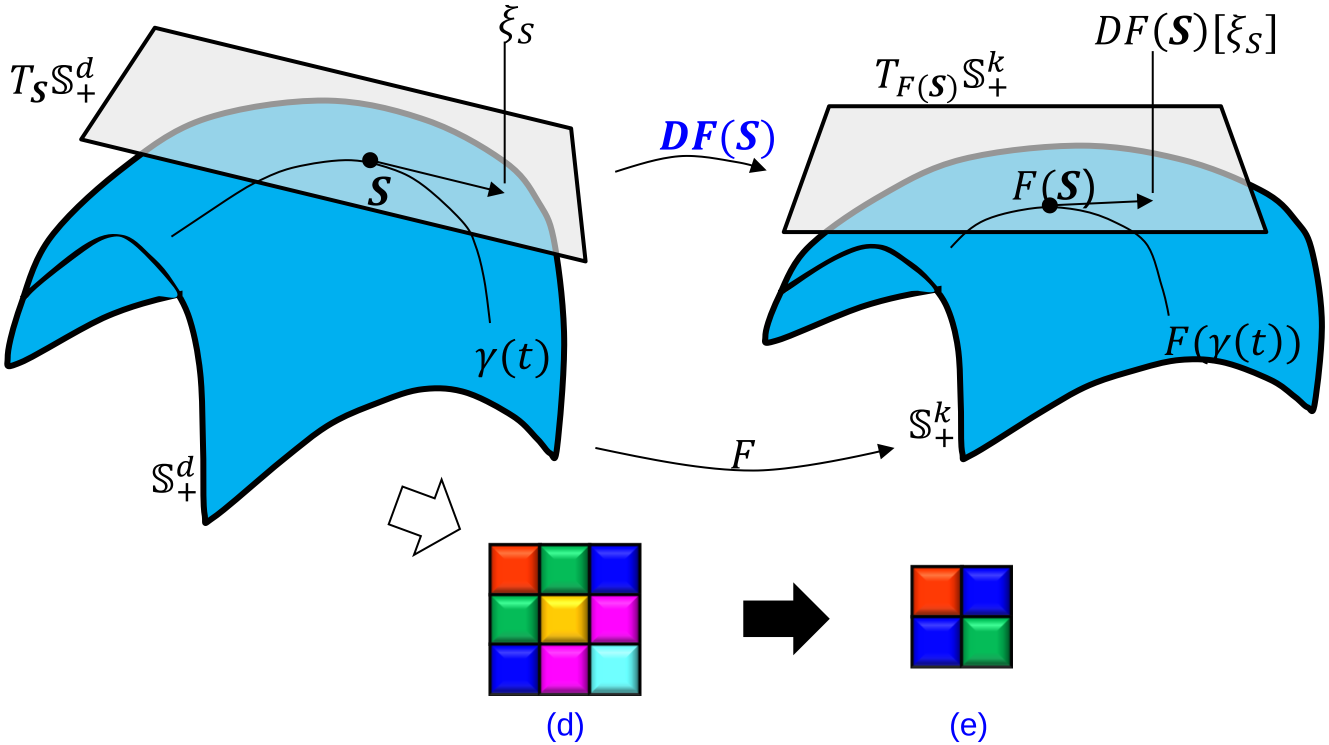


- Tangent space approximation ((a)-(b))
  - e.g., Tosato *et al.* 2010, Carreira *et al.* 2012, Vemulapalli *et al.* 2015
- Hilbert space embedding ((a)-(c)-(b))
  - e.g., Wang *et al.* 2012, Jayasumana *et al.* 2013, Minh *et al.* 2014

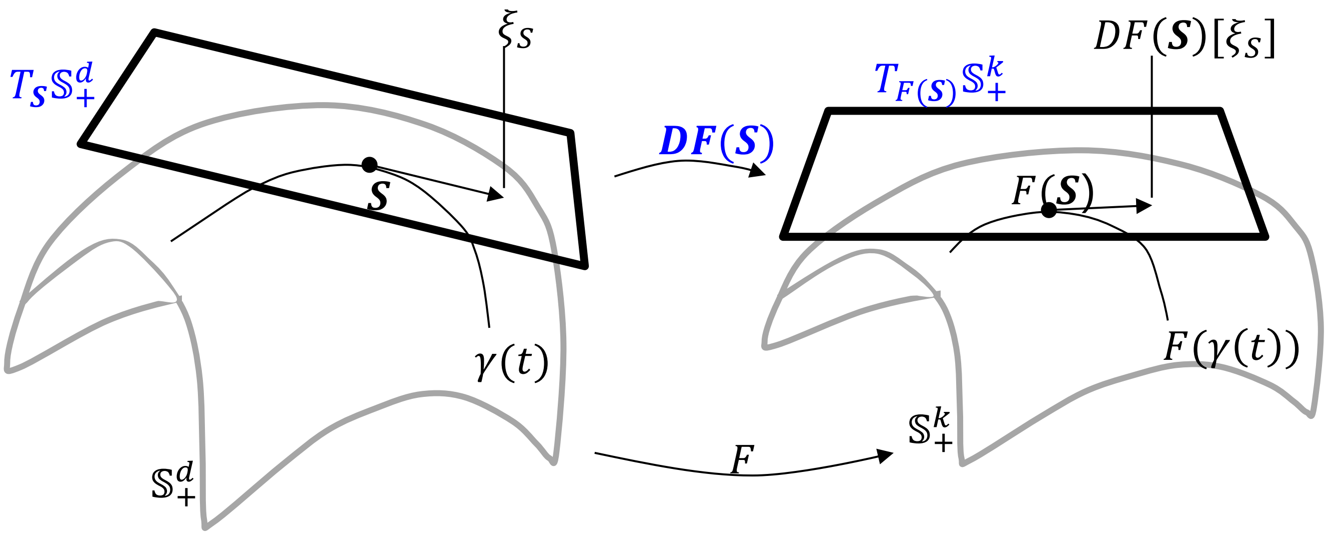
- Convert SPD matrix logarithm into vector-form in tangent space at identity matrix ((a)-(b1)/(b2))
  - Ignore the symmetric property of SPD matrix logarithm
  - Work inefficiently on the SPD vector-form often of high dimensionality



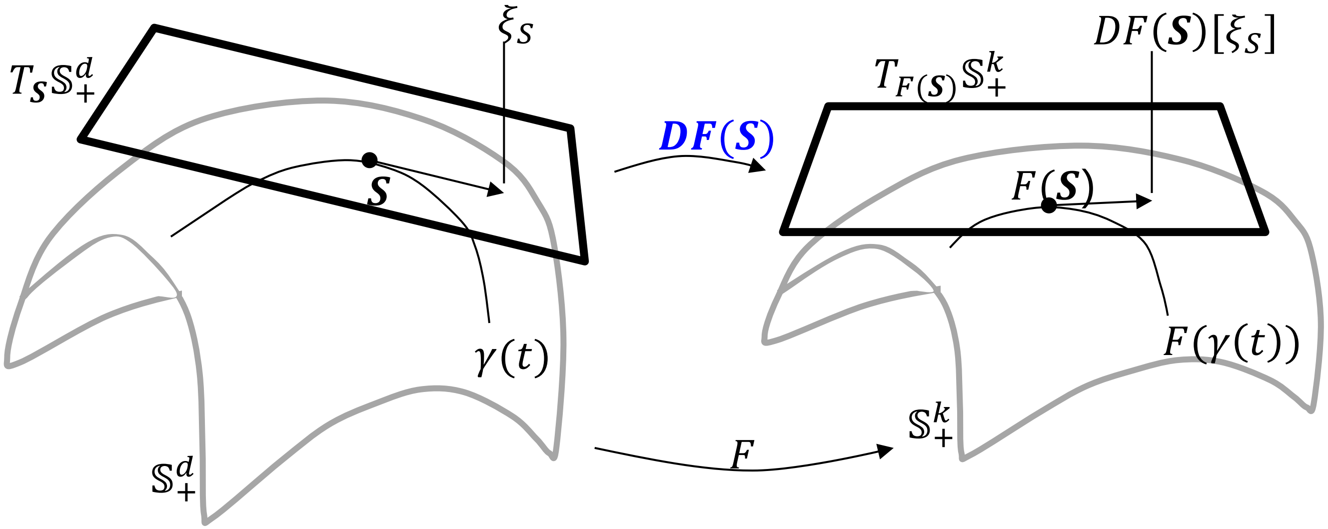
Our Approach



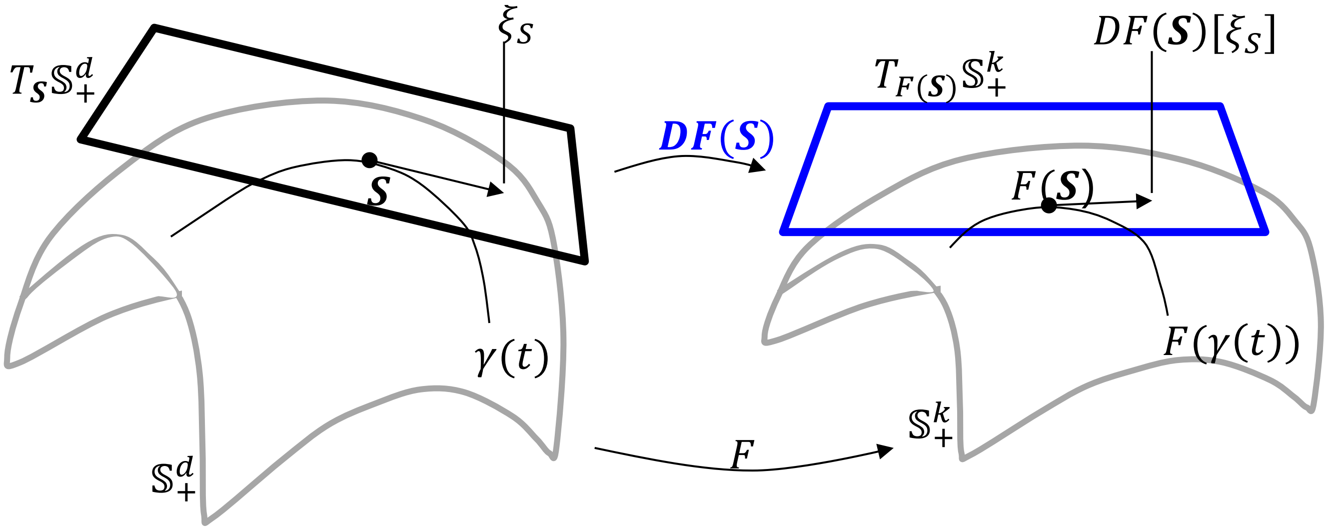
- Learn tangent-to-tangent map $\boldsymbol{DF(S)}$
  - Work on the matrix-form of SPD matrix logarithm ((d)-(e))
    - Keep the symmetric property of SPD matrix logarithm
    - Work efficiently on lower-dimensional matrix-form



- Learn tangent-to-tangent map
  - From original tangent space $T_{\boldsymbol{S}}\mathbb{S}_+^d$ to a more discriminant tangent space $T_{F(\boldsymbol{S})}\mathbb{S}_+^k$
    - $DF(\boldsymbol{S}): T_{\boldsymbol{S}}\mathbb{S}_+^d \rightarrow T_{F(\boldsymbol{S})}\mathbb{S}_+^k$
    - *If $DF(\boldsymbol{S})$ is an injection, the manifold-to-manifold map $F: \mathbb{S}_+^d \rightarrow \mathbb{S}_+^k$ is an immersion*



- Learn tangent-to-tangent map
  - Specific form of tangent map
    - $DF(\boldsymbol{S})$: $f(\log(\boldsymbol{S})) = \boldsymbol{W}^T\log(\boldsymbol{S})\boldsymbol{W}$
      - $\log(\boldsymbol{S}) \in \mathbb{R}^{d\times d}, \boldsymbol{W} \in \mathbb{R}^{d\times k}, f(\log(\boldsymbol{S})) \in \mathbb{R}^{k\times k}$
      - if $\boldsymbol{W}$: column full rank, $f(\log(\boldsymbol{S}))$ *yields a valid symmetric matrix*



- Log-Euclidean metric on new SPD manifold
  - $d_l^2\big(f(\boldsymbol{T}_i), f(\boldsymbol{T}_j)\big) = ||\boldsymbol{W}^T\boldsymbol{T}_i\boldsymbol{W} - \boldsymbol{W}^T\boldsymbol{T}_j\boldsymbol{W}||_F^2$
    $= tr(\boldsymbol{Q}(\boldsymbol{T}_i - \boldsymbol{T}_j)(\boldsymbol{T}_i - \boldsymbol{T}_j))$
  - $\boldsymbol{T}_i = \log(\boldsymbol{S}_i), \boldsymbol{T}_j = \log(\boldsymbol{S}_j)$
  - $\boldsymbol{Q} = (\boldsymbol{W}\boldsymbol{W}^T)^2$: PSD matrix

*$\boldsymbol{W}\boldsymbol{W}^T(\boldsymbol{T}_i - \boldsymbol{T}_j)$ is required to be symmetric

- Objective function (matrix-form of the ITML method [Davis *et al.*, 2007])
  - $\arg\min_{\boldsymbol{Q},\xi} D_{\ell d}(\boldsymbol{Q}, \boldsymbol{Q}_{\mathbf{0}}) + \eta D_{\ell d}(\boldsymbol{\xi}, \boldsymbol{\xi}_{\mathbf{0}})$

    $\mathrm{s.t.}, \mathrm{tr}\left(\boldsymbol{Q}\boldsymbol{A}_{ij}^{T}\mathbf{A}_{ij}\right) \le \boldsymbol{\xi}_{c(i,j)}, c(i,j) \in \boldsymbol{S}$

    $\mathrm{tr}\left(\boldsymbol{Q}\boldsymbol{A}_{ij}^{T}\mathbf{A}_{ij}\right) \ge \boldsymbol{\xi}_{c(i,j)}, c(i,j) \in \boldsymbol{D}$
  - $D_{\ell d}$: LogDet divergence, $\mathbf{A}_{ij} = \log(\boldsymbol{C}_i) - \log(\boldsymbol{C}_j)$,
  - $\boldsymbol{S}/\boldsymbol{D}$: constraint set involving sample pairs with the same /different label(s)

- Optimization algorithm
  - Cyclic Bregman projection algorithm [Bregman,1967; Censor & Zenior, 1997]
    - Choose one constraint per iteration
    - Perform a projection so that the current solution satisfies the chosen constraint

Evaluated Methods

Method	Literature source	abbr.
SPD basic metric	Pennec *et al*., IJCV’2006	AIM
	Sra *et al*., NIPS’2012	Stein
	Arsigny *et al*., SIAM MAA’2007	LEM
SPD metric learning	Harandi *et al*., ECCV’2014	SPDML-AIM/Stein
	Harandi *et al*., ECCV’2012	RSR-Stein
	Wang *et al*., CVPR’2012	CDL-LEM
	Vemulapalli *et al*., arXiv’2015	ITML-LEM

- SPDML-AIM/Stein: SPD manifold learning (SPDML) with Affine-Invariant metric (AIM) or Stein divergence
- RSR-Stein: Riemannian Sparse Representation (RSR) with Stein divergence
- CDL-LEM: Covariance Discriminative Learning (CDL) with Log-Euclidean metric (LEM)
- ITML-LEM: **Information-Theoretic Metric Learning (ITML)** on vector-form of SPD matrix logarithm with **Log-Euclidean Metric (LEM)**

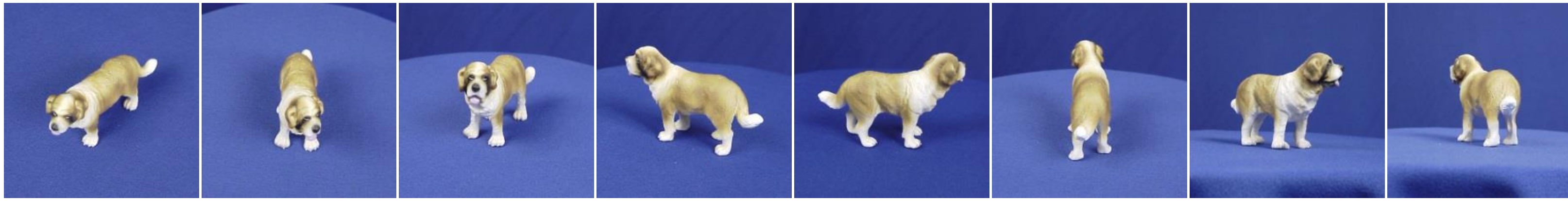
Set-based Object Categorization

- ETH-80 dataset (Leibe & Schiele, 2003)
  - 80 image sets of 8 object categories
    - Each category has 10 image sets
  - 20×20 resized intensity images
  - 401×401 SPD feature
  - Random selection for 10 tests
    - 50% for gallery, 50% for probe

$$\boldsymbol{S} = |\widetilde{\boldsymbol{C}}|^{-\frac{1}{d+1}} \begin{bmatrix} \widetilde{\boldsymbol{C}} + \widetilde{\boldsymbol{m}}\widetilde{\boldsymbol{m}}^T & \widetilde{\boldsymbol{m}} \\ \widetilde{\boldsymbol{m}}^T & 1 \end{bmatrix}$$

$\widetilde{\boldsymbol{C}}$: covariance matrix, $\widetilde{\boldsymbol{m}}$: mean

Set-based Object Categorization: Results



Method	Accuracy
AIM	87.50 ± 5.77
Stein	88.00 ± 5.11
LEM	89.25 ± 4.72
SPDML-AIM	90.75 ± 3.34
SPDML-Stein	90.50 ± 3.87
RSR-Stein	93.25 ± 3.34
CDL-LEM	93.75 ± 3.43
ITML-LEM	93.75 ± 3.43
LEML	$\mathbf{94.75 \pm 2.49}$
LEML-CDL	$\mathbf{96.00 \pm 2.11}$

Video-based Face Identification

- YouTube Celebrities dataset (Kim *et al.*, 2008)
  - 1,910 video sequences of 47 subjects from YouTube
    - Highly compressed, low resolution
  - 20×20 resized intensity images
  - 401×401 SPD feature
  - Random selection for 10 tests
    - 3 of 9 for gallery, 6 of 9 for probe

$$\boldsymbol{S} = |\widetilde{\boldsymbol{C}}|^{-\frac{1}{d+1}} \begin{bmatrix} \widetilde{\boldsymbol{C}} + \widetilde{\boldsymbol{m}}\widetilde{\boldsymbol{m}}^T & \widetilde{\boldsymbol{m}} \\ \widetilde{\boldsymbol{m}}^T & 1 \end{bmatrix}$$

$\widetilde{\boldsymbol{C}}$: covariance matrix, $\widetilde{\boldsymbol{m}}$: mean

Video-based Face Identification: Results

Method	Accuracy
AIM	62.85±3.46
Stein	61.46±3.53
LEM	63.91±3.25
SPDML-AIM	64.66±2.92
SPDML-Stein	61.57±3.43
RSR-Stein	72.77±2.69
CDL-LEM	72.67±2.47
ITML-LEM	66.51±3.67
LEML	**70.53±2.95**
LEML-CDL	**73.31±2.49**

Video-based Face Verification

- YouTube Faces DB (Wolf *et al.*, 2011)
  - 3,425 video sequences of 1,595 subjects from YouTube
    - Highly compressed, low resolution
  - 24×40 resized intensity images
  - 961×961 SPD feature
  - Random selection for 10 folds
    - 9 folds for training, 1 fold for testing

$$\boldsymbol{S} = |\widetilde{\boldsymbol{C}}|^{-\frac{1}{d+1}} \begin{bmatrix} \widetilde{\boldsymbol{C}} + \widetilde{\boldsymbol{m}}\widetilde{\boldsymbol{m}}^T & \widetilde{\boldsymbol{m}} \\ \widetilde{\boldsymbol{m}}^T & 1 \end{bmatrix}$$

$\widetilde{\boldsymbol{C}}$: covariance matrix, $\widetilde{\boldsymbol{m}}$: mean

Video-based Face Verification: Results

Method	Accuracy
AIM	59.28±2.25
Stein	58.70±1.97
LEM	61.48±2.27
SPDML-AIM	62.16±2.16
SPDML-Stein	62.56±2.49
RSR-Stein	N/A
CDL-LEM	66.76±1.89
ITML-LEM	60.02±1.84
LEML	**65.12±1.54**
LEML-CDL	**72.34±2.07**

Running Time

- Training and testing (classification of one video) time on YouTube Celebrities dataset

Method	Train	Test
SPDML-AIM	15072.56	9.35
SPDML-Stein	108.50	0.04
ITML-LEM	92007.13	0.02
LEML	**56.30**	**0.02**

Conclusions

- Our approach seeks to map the SPD matrix logarithms from the original tangent space to a more discriminant tangent space
- This keeps the symmetric property of SPD matrix logarithms, and works effectively on matrix-form

- Future work:
  – Study if the proposed SPD metric learning could be extended to end-to-end SPD feature learning
    » leverage the existing deep learning technique

Thank you!